Considerations for Effective Fractions Instruction

Yukari Okamoto
Professor of Education
University of California, Santa Barbara

2009 NAEP 4th grade item

Which fraction has a value closest to 1/2?

a) 5/8 → 25%

b) 1/6 → 6%

c) 2/2 → 41%

d) 1/5 → 26%

3rd grade Common Core SS

Recognize and generate simple equivalent fractions

e.g., 1/2 = 2/4 = 4/8

http://nces.ed.gov/nationsreportcard/itmrlsx/detail.aspx?subject=mathematics

Why are fractions difficult?

- What students know about whole numbers does not always hold up…for example,
  - Multiplying does not always produce larger numbers
  - Dividing does not always produce smaller numbers.

Why are fractions difficult?

- Properties of fractions are new and unfamiliar…for example,
  - A fraction refers to a specific quantity and is not made up of two whole numbers.
  - There is an infinite number of numbers between any two adjacent whole numbers.
  - Each fraction can be expressed in many ways.

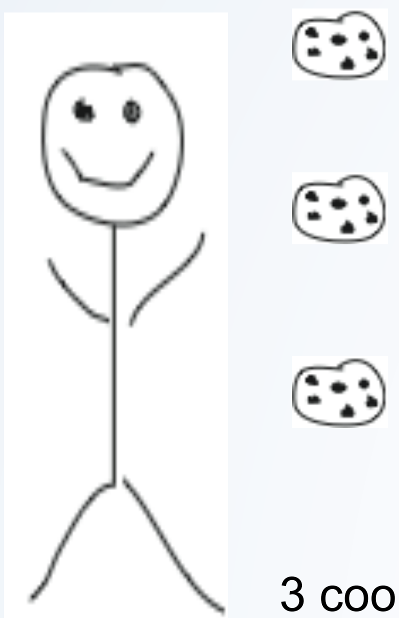
Recommendation 1

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts

- ✓ Use *equal-sharing activities* to introduce the concept of fractions.
- ✓ Use sharing activities that involve dividing sets of objects as well as single whole objects.

Sharing Cookies

Two children want to share 6 cookies so that each child gets the same amount to eat.

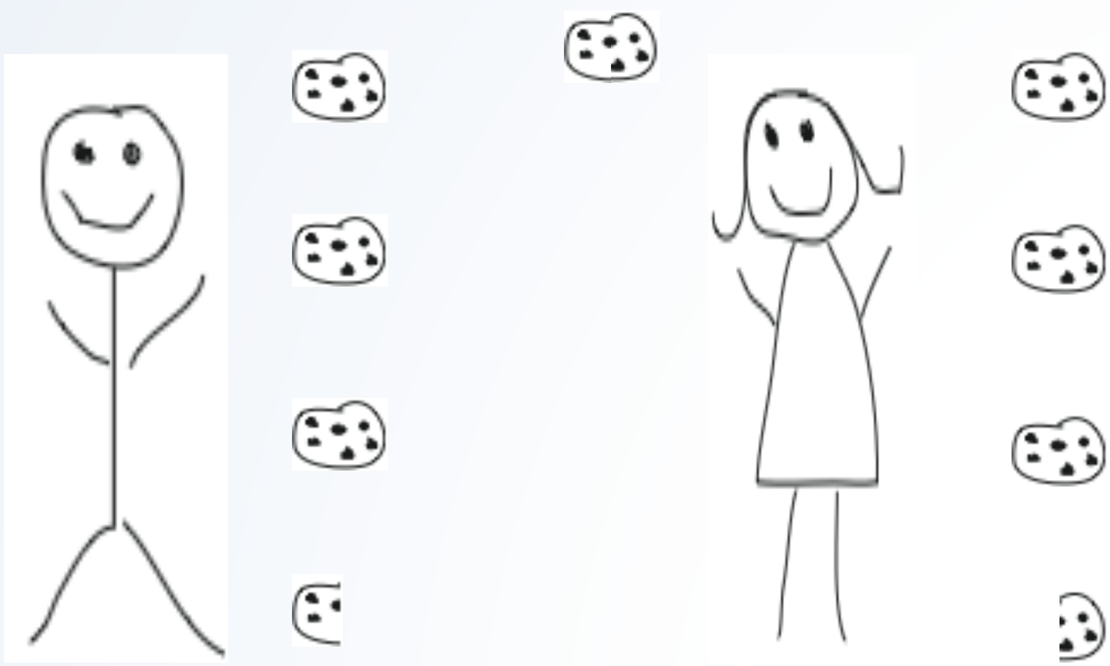


3 cookies
-> 1/2 of total

3 cookies
-> 1/2 of total

Sharing Cookies

Two children want to share **7** cookies so that each child gets the same amount to eat.

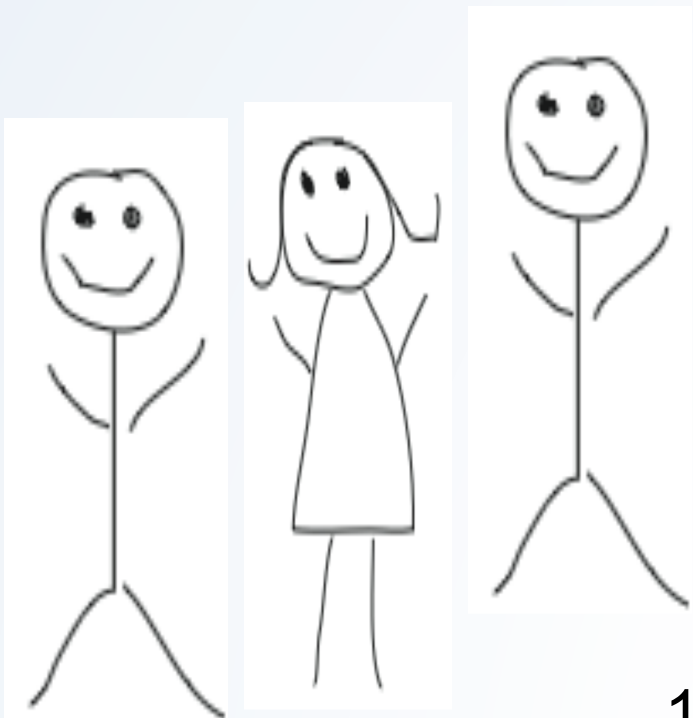


3 1/2 cookies
-> 1/2 of total

3 1/2 cookies
-> 1/2 of total

Not just discrete items…

Two children want to share a candy bar so that each child gets the same amount to eat.



3 children?

1/3 each?

1/2 each

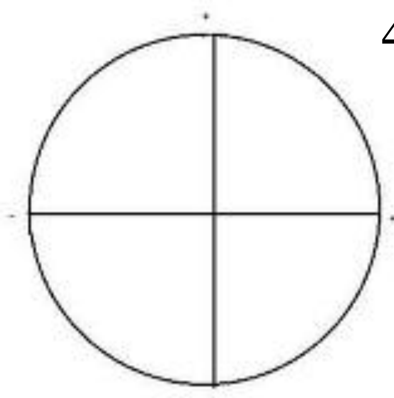
Recommendation 1

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts

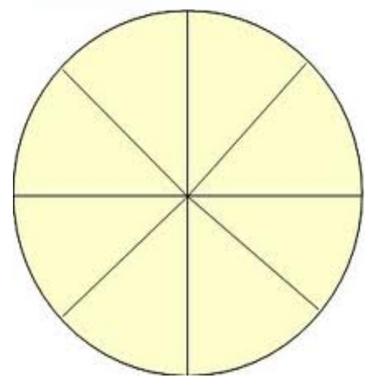
✓ Extend equal-sharing activities to develop students' understanding of ordering and *equivalence of fractions*.

– Extend equal-sharing activities to develop students' understanding of ordering and *equivalence of fractions*.

One pizza is cut to 4 slices. Another pizza is cut into 8 slices.



$$\frac{1}{4} \neq \frac{1}{8}$$



$$\frac{1}{4} = \frac{2}{8}$$

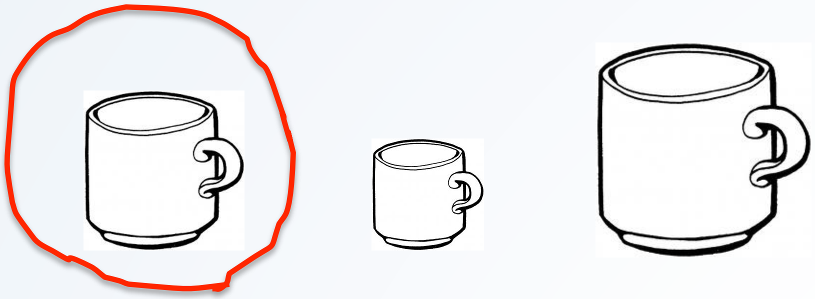
Recommendation 1

Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts

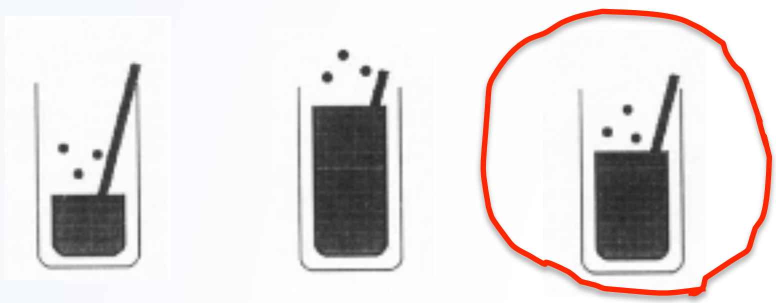
- ✓ Build on students' informal understanding of proportionality to develop more advanced understanding of *proportional reasoning*.
- ✓ Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.

Early Proportional Reasoning 1a

Experimenter



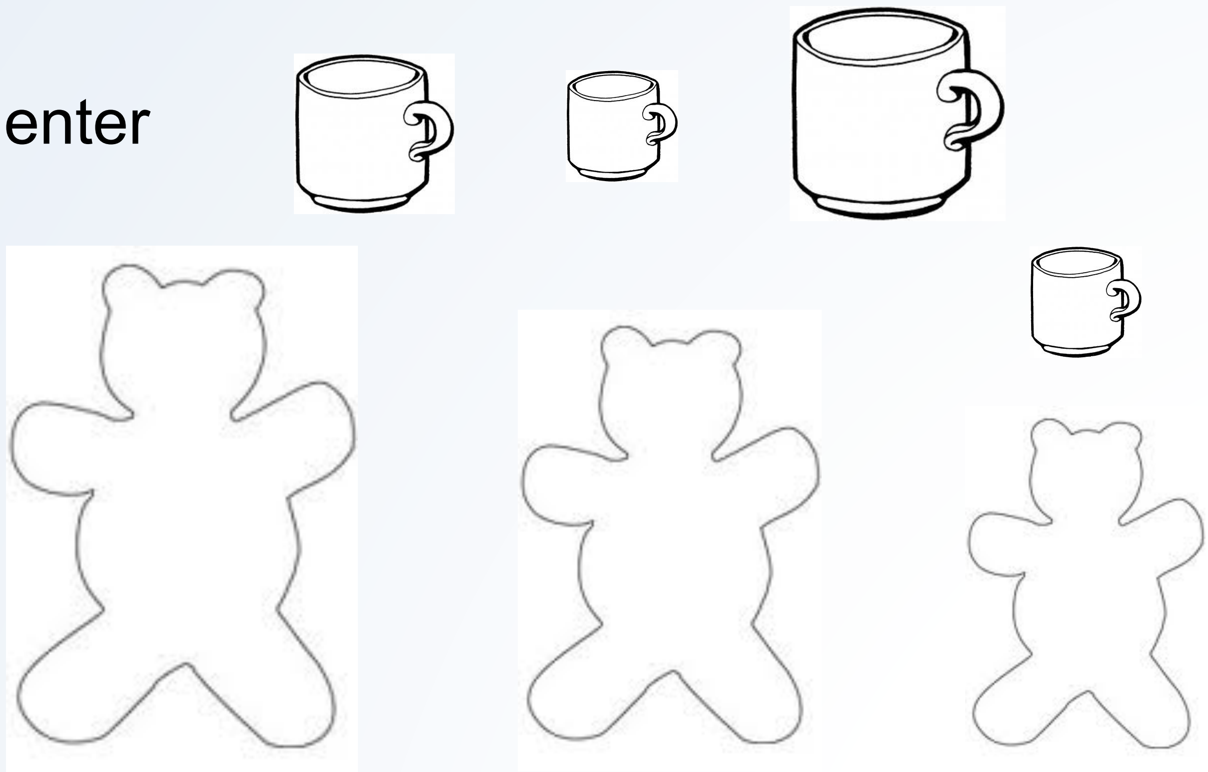
Child



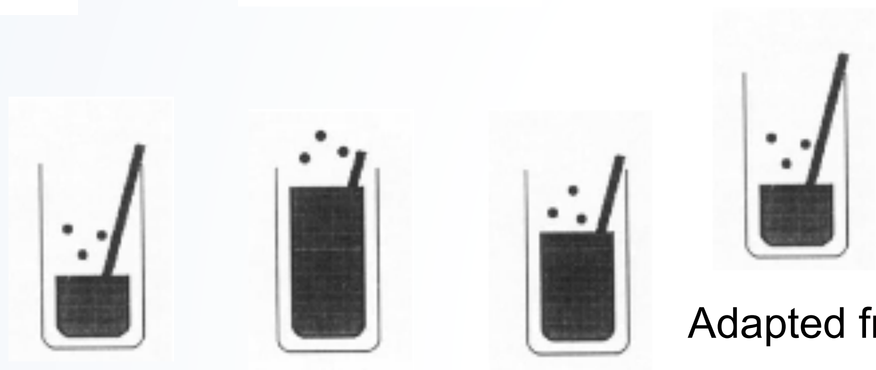
Adapted from Goswami (1995)

Early Proportional Reasoning 1b

Experimenter

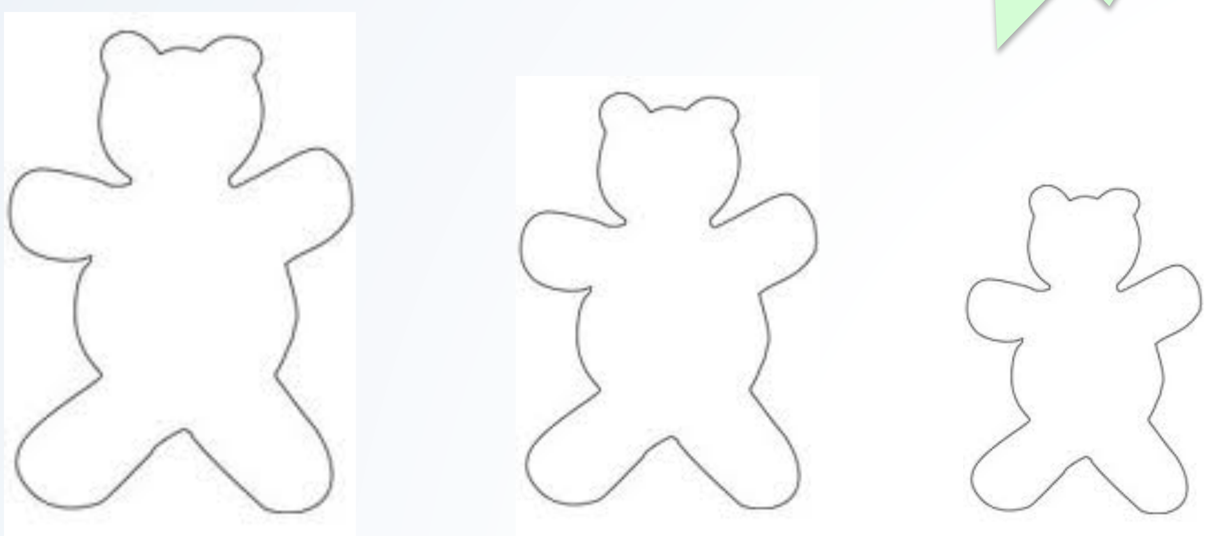


Child's task



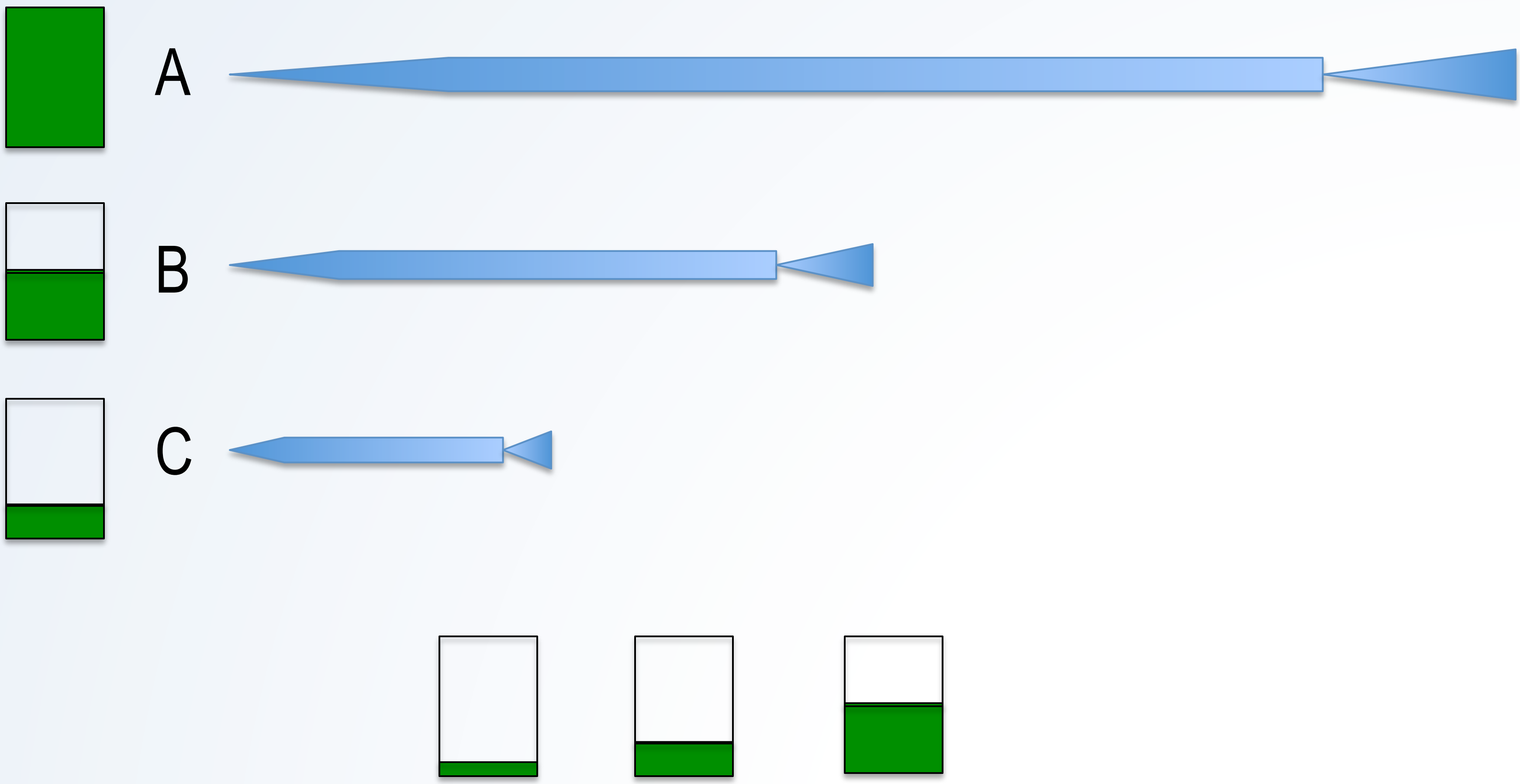
Adapted from Goswami (1995)

Early Proportional Reasoning 1c



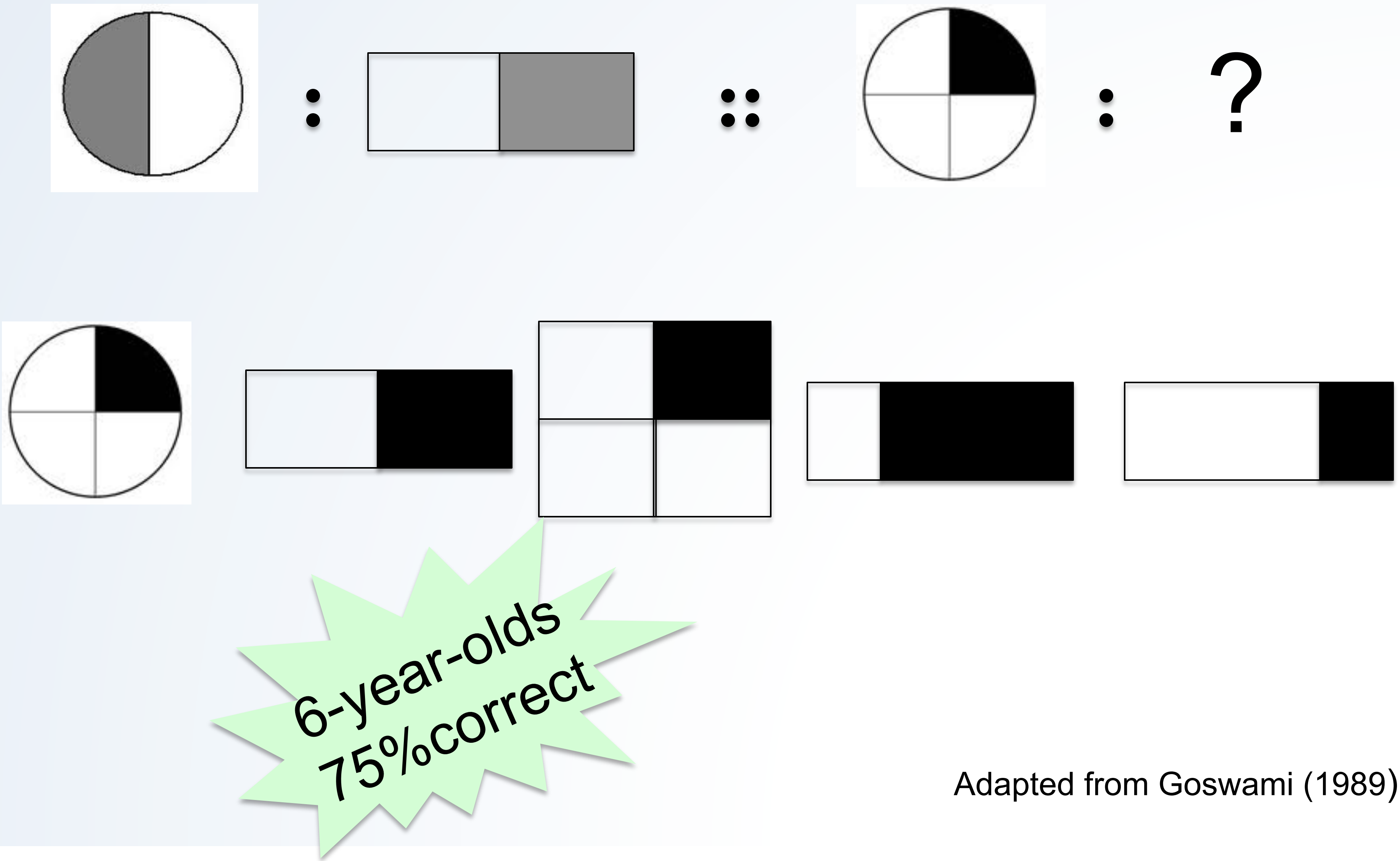
Adapted from Goswami (1995)

Early Proportional Reasoning 2



Chen & Okamoto (1998)

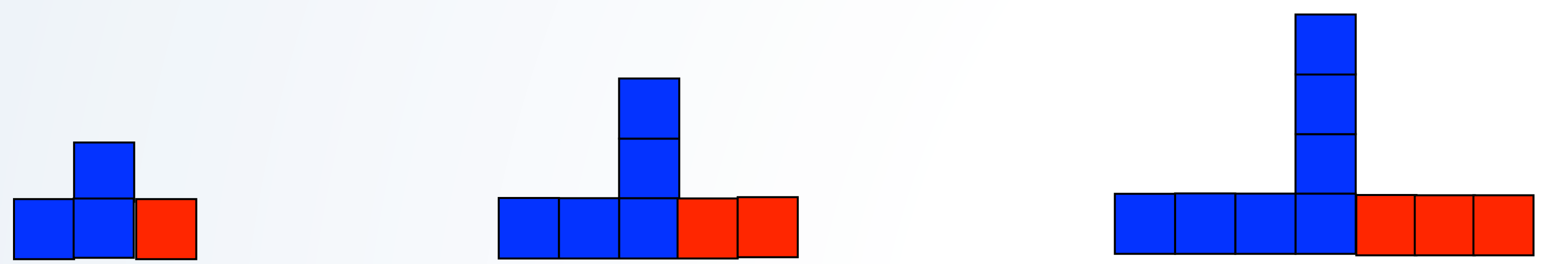
Early Proportional Reasoning 3



Adapted from Goswami (1989)

Key Ideas…

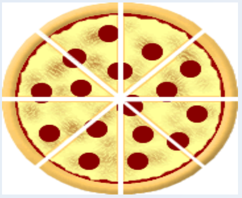
- Proportional relations
- Covariation
- Patterns



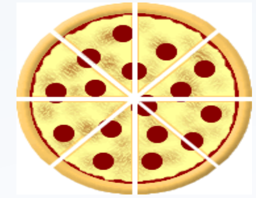
Recommendation 2

- **Help students recognize that *fractions are numbers* and that they expand the number system beyond whole numbers.**

- **Use *number lines* as a central representational tool in teaching this and other fraction concepts from the early grades onward.**



How much is 1/8 + 5/8 = ?



- Interviewer: Melanie, these two pizzas are each cut into eight slices for a party. This pizza on the left has seven pieces eaten from it. How much pizza is left there?
- **Melanie:** ***One-eighth, writes 1/8.***
- Interviewer: The pizza on the right had three pieces eaten from it. How much is left of that pizza ?
- **Melanie:** ***Five-eighths, writes 5/8.***
- Interviewer: If you put those two together, how much of a pizza is left?
- **Melanie:** ***Six-eighths, writes 6/8.***
- Interviewer: Could you write a number sentence to show what you just did?
- **Melanie:** ***Writes 1/8 + 5/8 = 6/16.***
- Interviewer: That's not the same as you told me before. Is that OK?
- **Melanie:** ***Yes, this is the answer you get when you add fractions.***

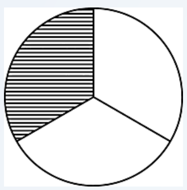
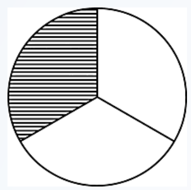
There are 6 cookies.
One is a chocolate cookie.
Write a fraction for the relation of the chocolate cookie to all cookies.

$$\frac{\boxed{1}}{\boxed{6}}$$

Help students recognize that fractions are numbers

Potential Misconception

How much is $\frac{1}{3}$ and $\frac{1}{3}$?

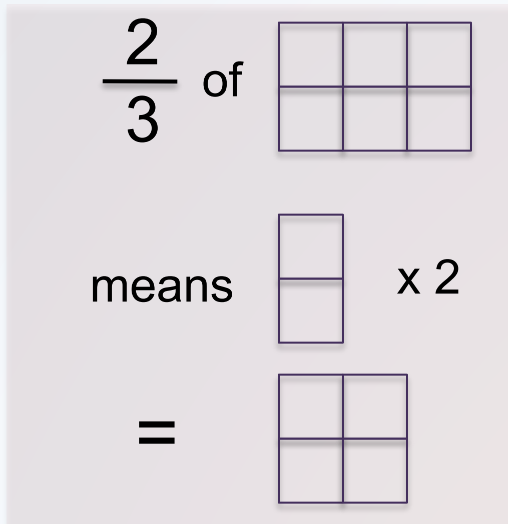
 + 

$$\frac{1}{3} + \frac{1}{3} = \frac{1+1}{3+3} = \frac{2}{6}$$???

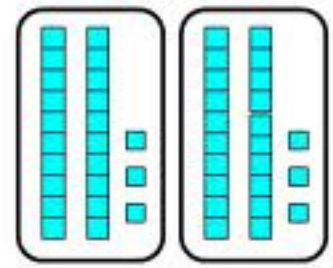
Part-whole isn't the only way to show a fraction…

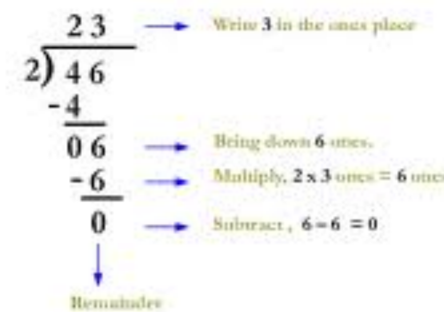
Part-whole

Operator

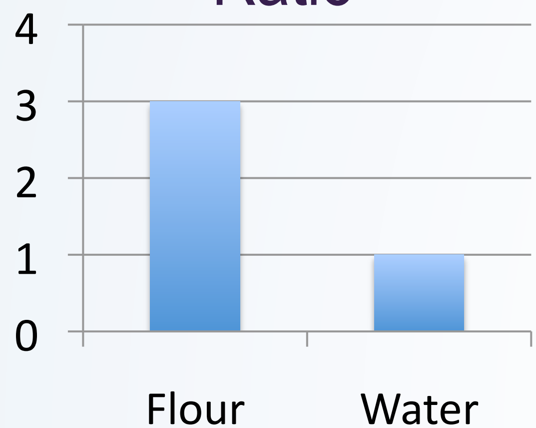


Quotient





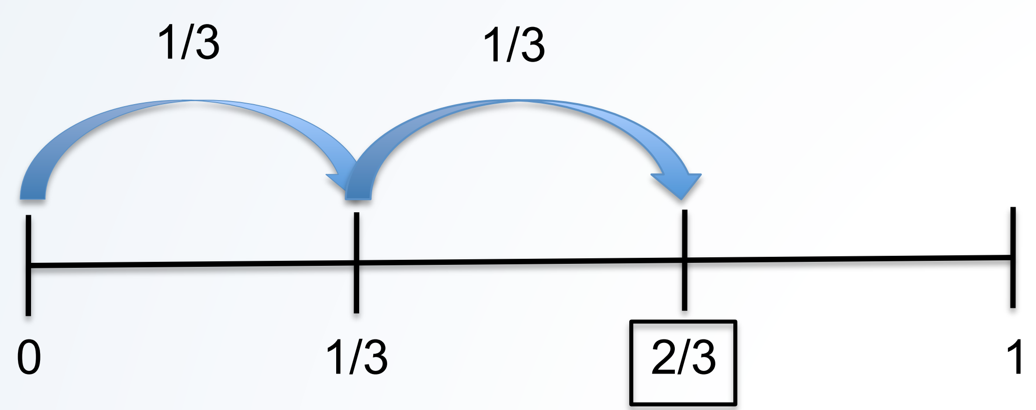
Ratio



Measure

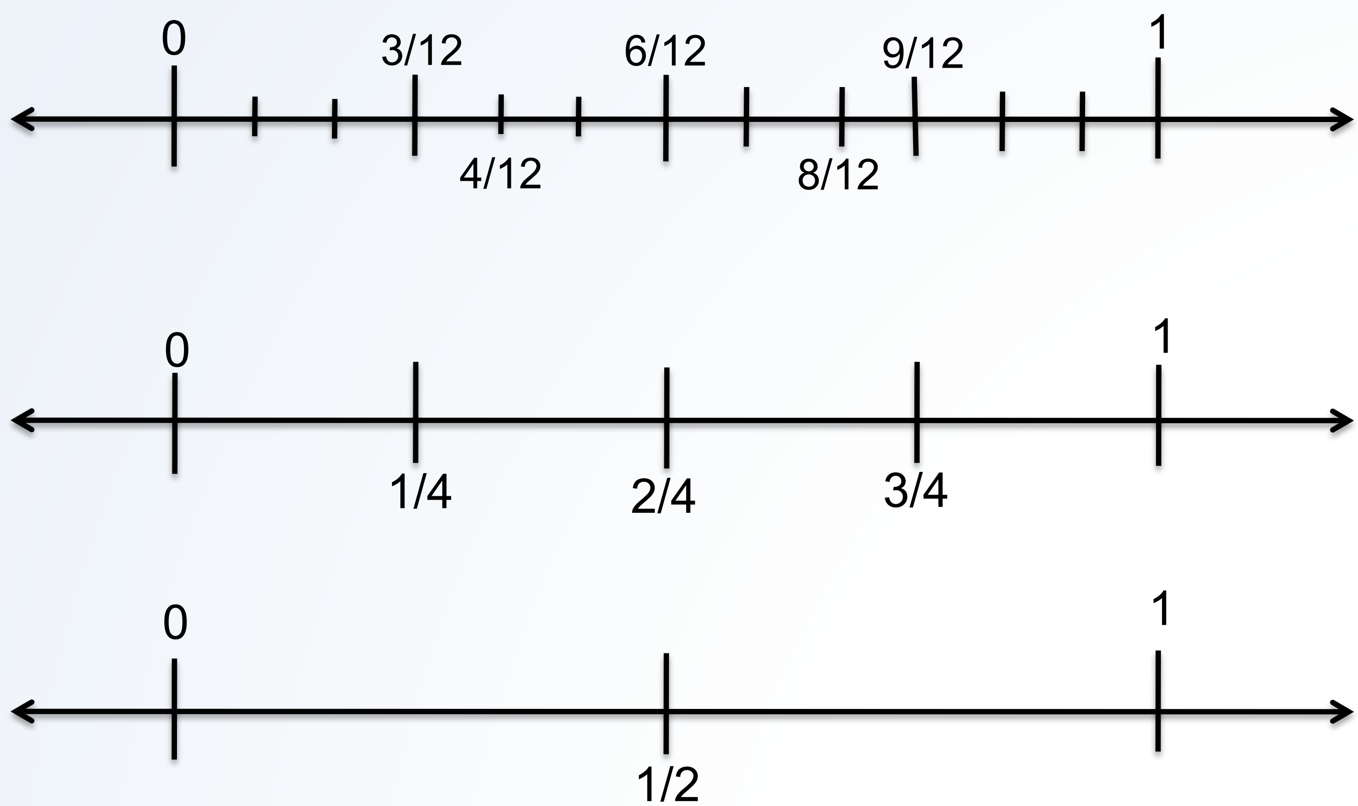
Help students recognize that fractions are numbers

How much is $\frac{1}{3}$ and $\frac{1}{3}$?

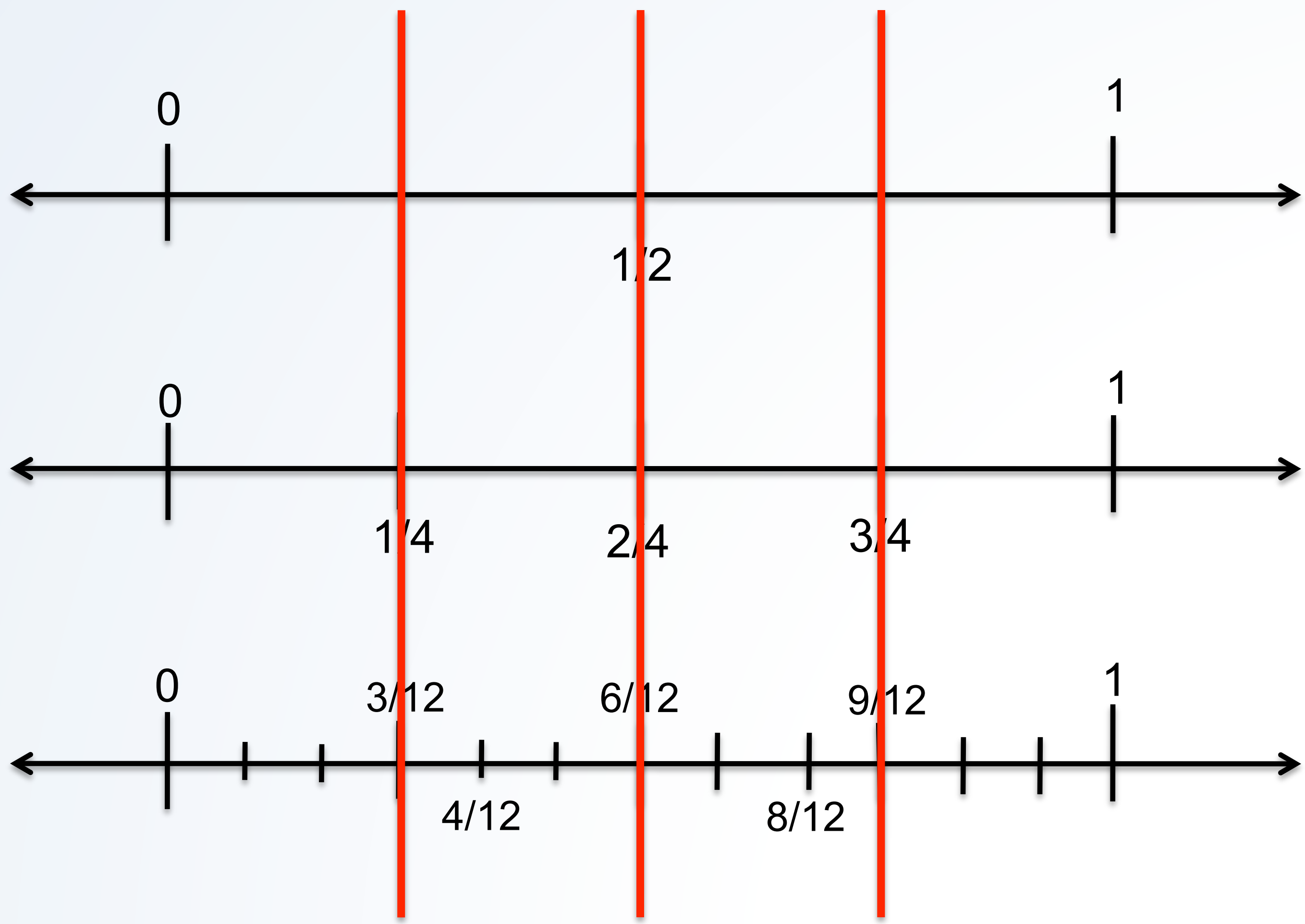


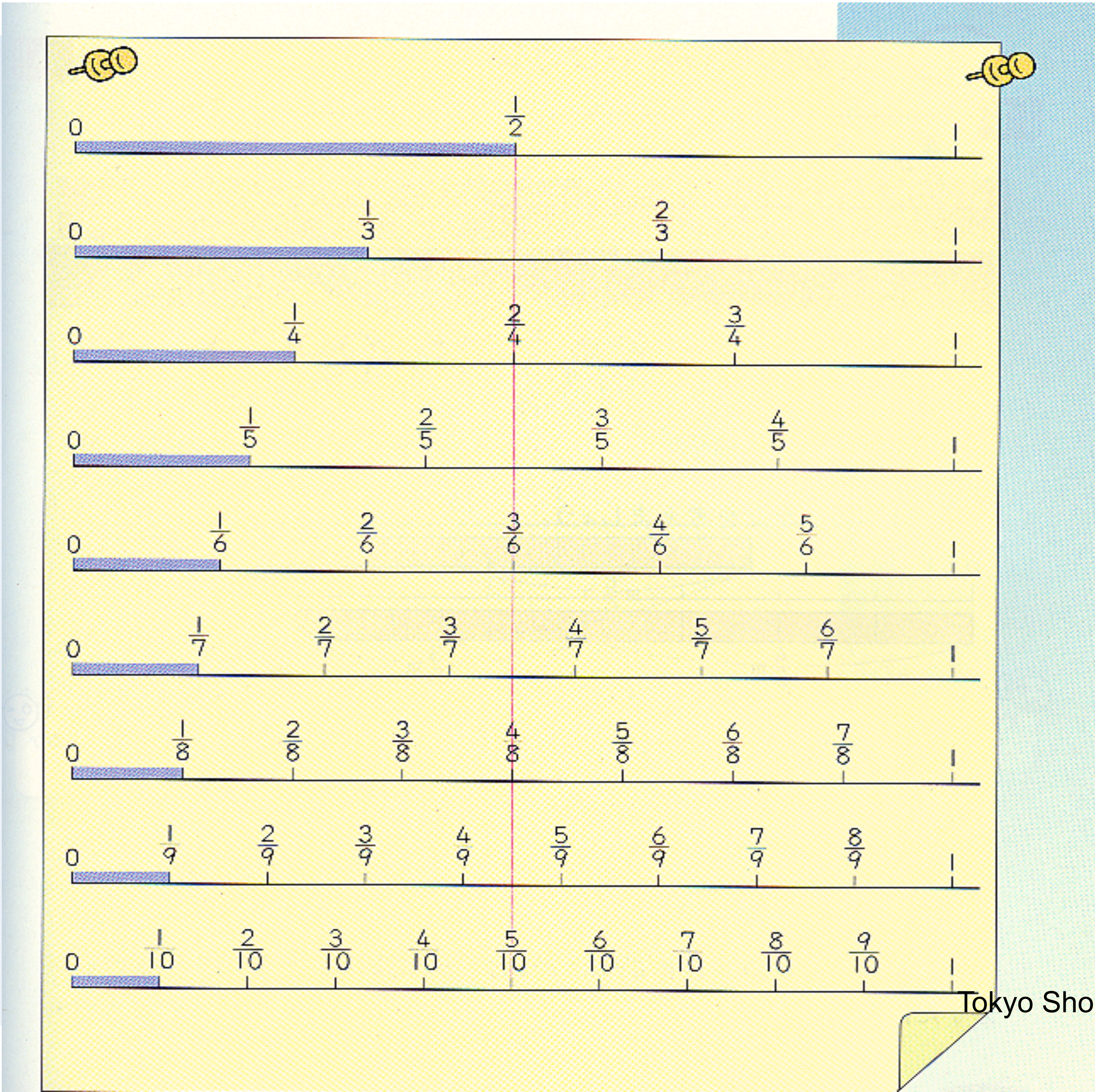
**** Number Lines are a useful tool to address this misconception ****

Number Line as a Representational Tool



Equivalent Fractions on a Number Line





Tokyo Shoseki (2010)

Recommendation 3

- **Help students understand why procedures for computations with fractions makes sense.**

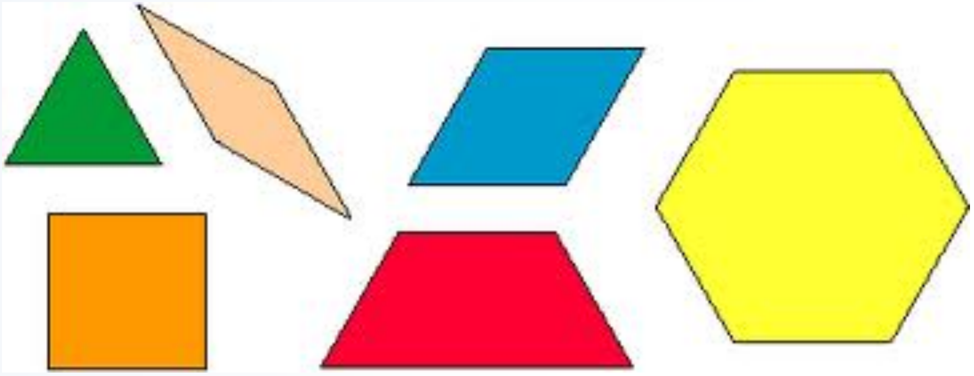
  - *Use area models, number lines, and other visual representations to improve students' understanding of formal computational procedures.*

  - *Provide opportunities for students to use estimation to predict or judge the reasonableness of answers to problems involving computation with fractions.*

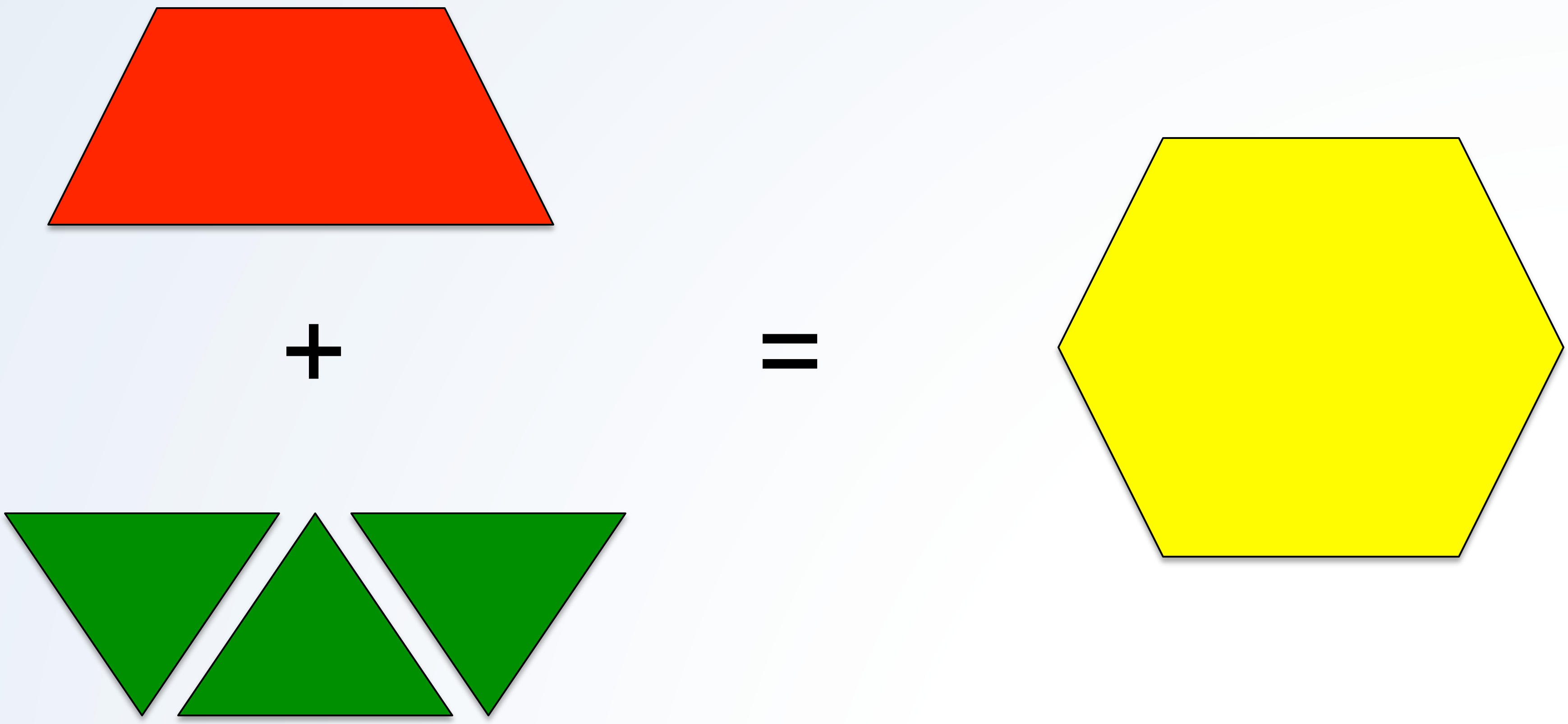
  - *Present real-world contexts with plausible numbers for problems that involve computing with fractions.*

  - *Address common misconceptions regarding computational procedures with fractions.*

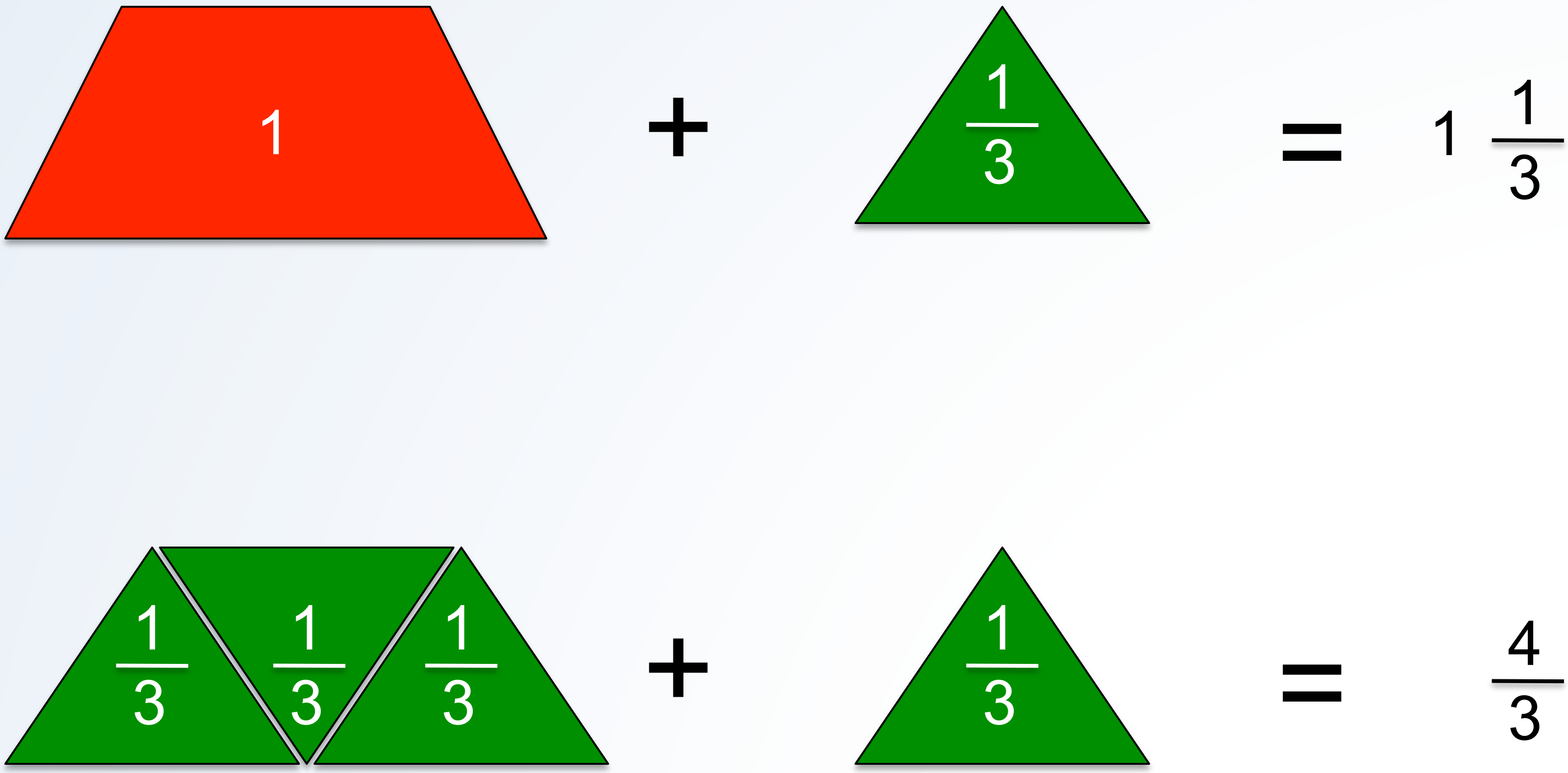
Pattern Blocks



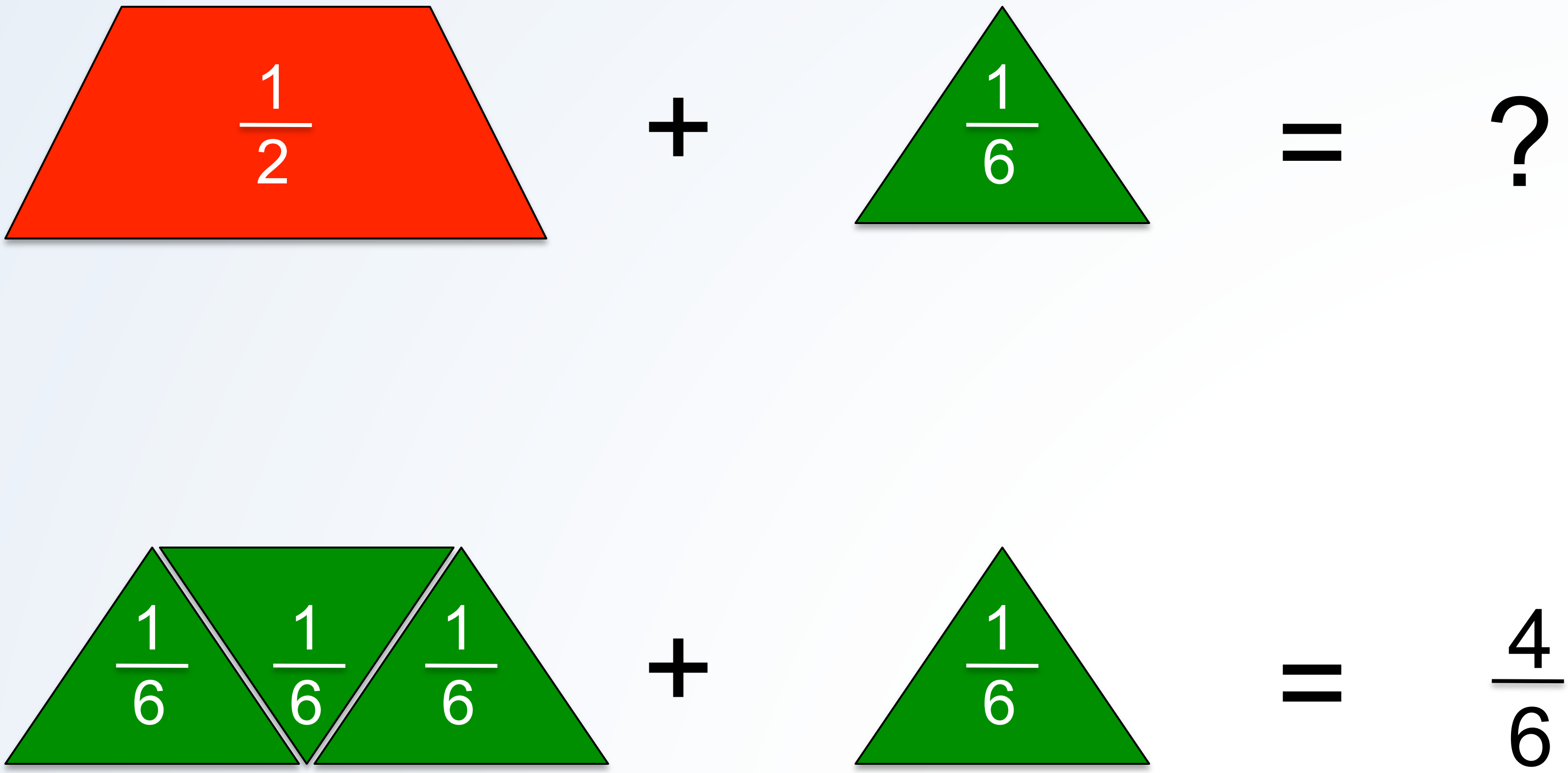
Pattern Blocks



Pattern Blocks

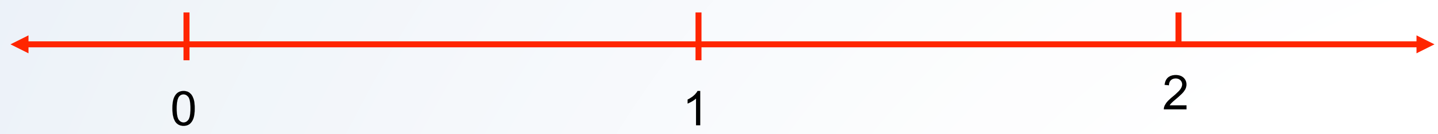


Pattern Blocks



Estimation

- Where would 2/3 + 1/6 be on this number line?



Sense Making:

2/3 is closer to 1 than to 0
adding 1/6 to that makes it a bit closer to 1
but it doesn't get to 1, so the sum should be less than 1

Use Meaningful, Real-World Examples

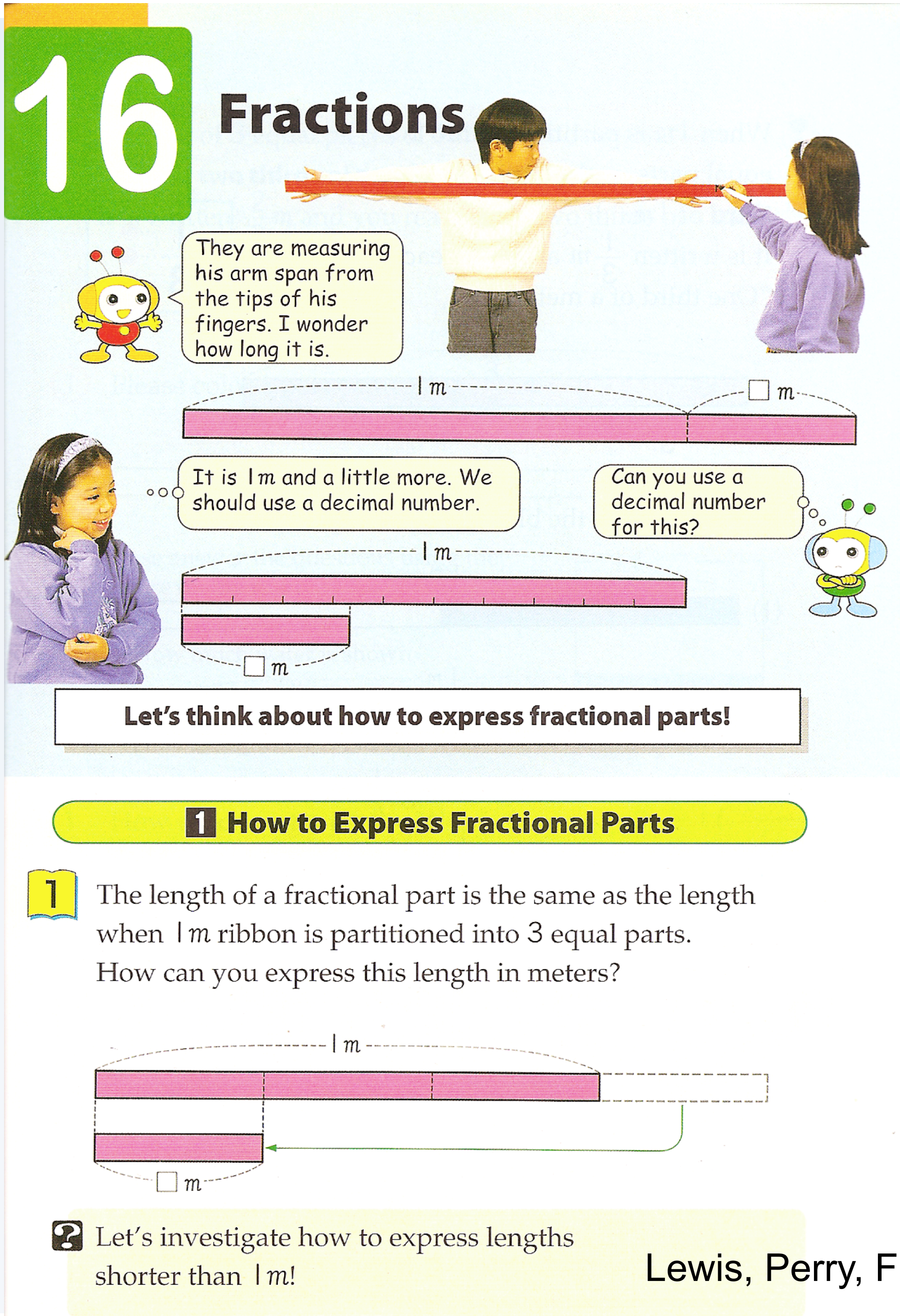
- There are 25 students in our class. Each student will get ¼ of a pizza. How many pizzas should we order?

1 pizza serves 4 students
6 pizzas serve 24 students
we need ¼ pizza for the last student
so 6 and ¼ pizzas!

No, you can't order 1/4 of a pizza

How many pizzas should we order?

7 pizzas



16 Fractions

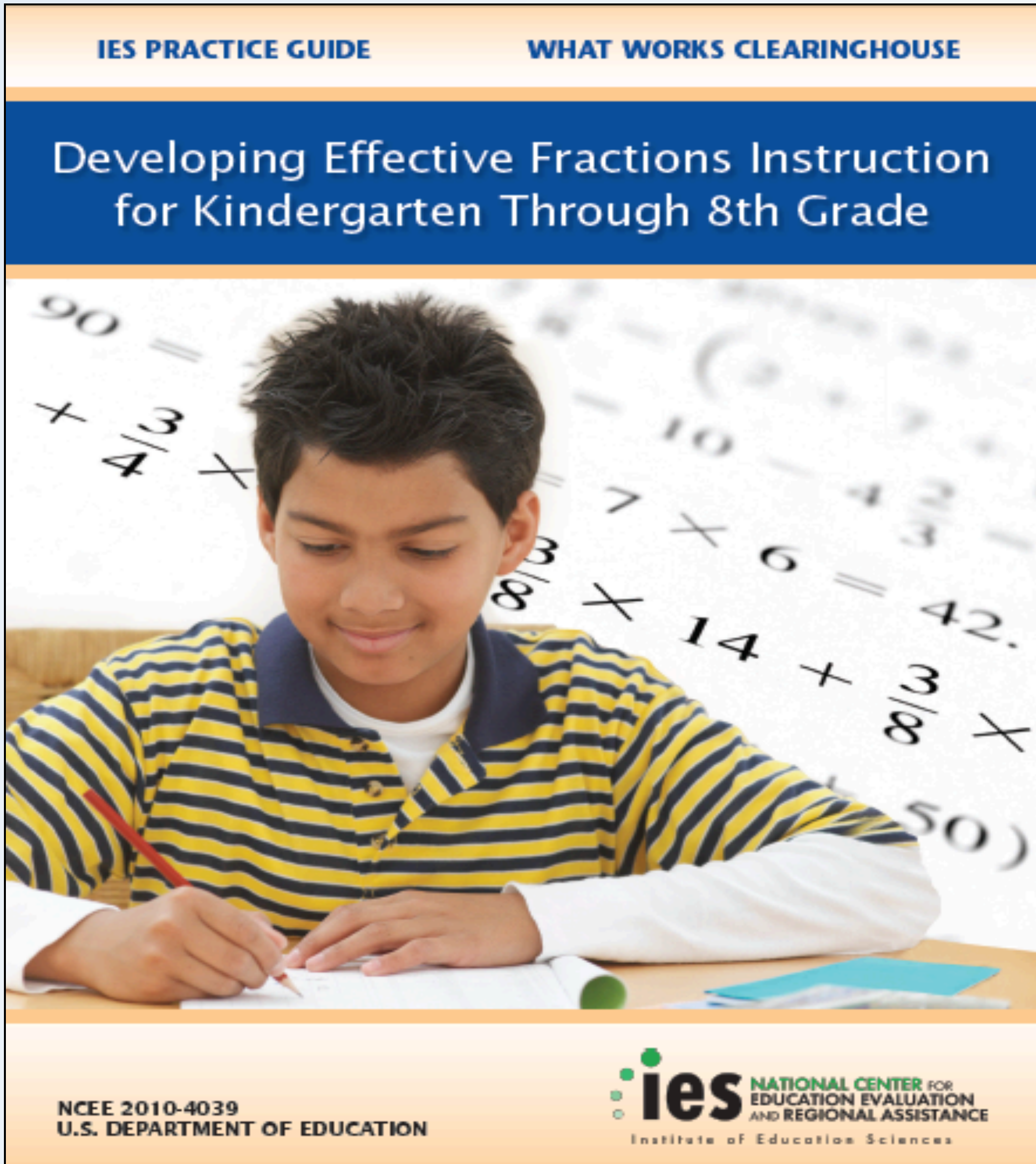
Let's think about how to express fractional parts!

1 How to Express Fractional Parts

1 The length of a fractional part is the same as the length when 1 m ribbon is partitioned into 3 equal parts. How can you express this length in meters?

Let's investigate how to express lengths shorter than 1 m!

Lewis, Perry, Friedkin, & Baker (2010)



http://ies.ed.gov/ncee/wwc/practiceguide.aspx?sid=15

Panel

Robert Siegler (Chair)
Carnegie Mellon University

Thomas Carpenter
University of Wisconsin-Madison

Francis (Skip) Fennell
McDaniel College

David Geary
University of Missouri at Columbia

James Lewis
University of Nebraska-Lincoln

Yukari Okamoto
University of California-Santa Barbara

Laurie Thompson
Elementary Teacher

Jonathan (Jon) Wray
Howard County (MD) Public Schools

Staff

Jeffrey Max
Andrew Gothro
Sarah Prenovitz
Mathematical Policy Research

Project Officer - **Susan Sanchez**
Institution of Education Sciences (IES)